LESSON

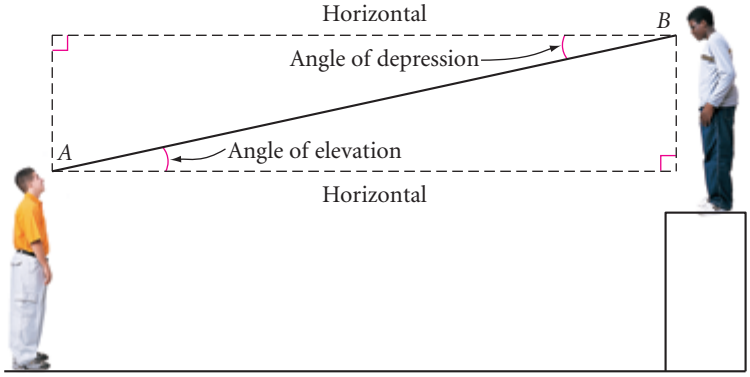
12.2

Problem Solving with Right Triangles

What science can there be more noble, more excellent, more useful . . . than mathematics?

BENJAMIN FRANKLIN

Right triangle trigonometry is often used indirectly to find the height of a tall object. To solve a problem of this type, measure the angle from the horizontal to your line of sight when you look at the top or bottom of the object.

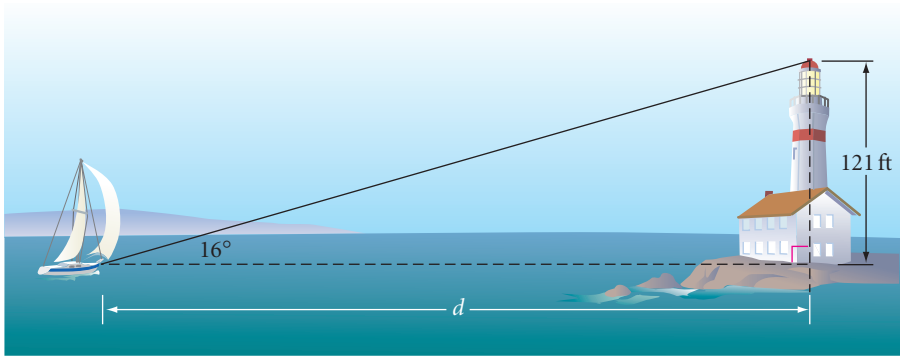


If you look up, you measure the **angle of elevation.** If you look down, you measure the **angle of depression.**

Here's an example.

EXAMPLE

The angle of elevation from a sailboat to the top of a 121-foot lighthouse on the shore measures 16°. To the nearest foot, how far is the sailboat from shore?



▶ Solution

The height of the lighthouse is opposite the 16° angle. The unknown distance is the adjacent side. Set up a tangent ratio.

$$\tan 16° = \frac{121}{d}$$

$$d(\tan 16°) = 121$$

$$d = \frac{121}{\tan 16°}$$

$$d \approx 422$$

The sailboat is approximately 422 feet from shore.

EXERCISES

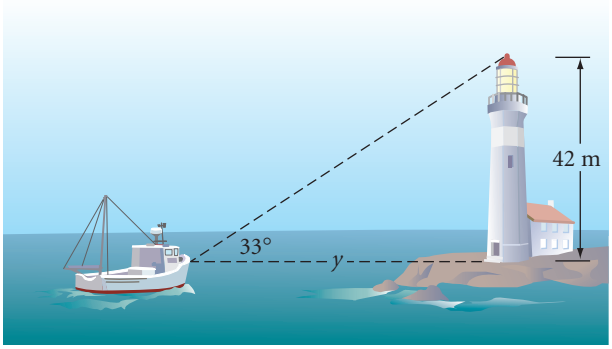
You will need

A calculator for Exercises **1–19**

1. According to a Chinese legend from the Han dynasty (206 B.C.E.–220 C.E.), General Han Xin flew a kite over the palace of his enemy to determine the distance between his troops and the palace. If the general let out 800 meters of string and the kite was flying at a 35° angle of elevation, how far away was the palace from General Han Xin's position?

2. Benny is flying a kite directly over his friend, Frank, who is 125 meters away. When he holds the kite string down to the ground, the string makes a 39° angle with the level ground. How high is Benny's kite?

3. **APPLICATION** The angle of elevation from a ship to the top of a 42-meter lighthouse on the shore measures 33°. How far is the ship from the shore? (Assume the horizontal line of sight meets the bottom of the lighthouse.)



4. **APPLICATION** A salvage ship's sonar locates wreckage at a 12° angle of depression. A diver is lowered 40 meters to the ocean floor. How far does the diver need to walk along the ocean floor to the wreckage?

5. **APPLICATION** A meteorologist shines a spotlight vertically onto the bottom of a cloud formation. He then places an angle-measuring device 65 meters from the spotlight and measures a 74° angle of elevation from the ground to the spot of light on the clouds. How high are the clouds?

The distance from the ground to a cloud formation is called the cloud *ceiling*.

6. **APPLICATION** Meteorologist Wendy Stevens uses a theodolite (an angle-measuring device) on a 1-meter-tall tripod to find the height of a weather balloon. She views the balloon at a 44° angle of elevation. A radio signal from the balloon tells her that it is 1400 meters from her theodolite.

a. How high is the balloon? (h)

b. How far is she from the point directly below the balloon?

c. If Wendy's theodolite were on the ground rather than on a tripod, would your answers change? Explain your reasoning.

Science CONNECTION

Weather balloons carry into the atmosphere what is called a *radiosonde,* an instrument with sensors that detect information about wind direction, temperature, air pressure, and humidity. Twice a day across the world, this upper-air data is transmitted by radio waves to a receiving station. Meteorologists use the information to forecast the weather.

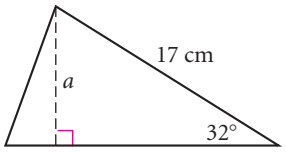
7. **APPLICATION** A ship's officer sees a lighthouse at a 42° angle to the path of the ship. After the ship travels 1800 m, the lighthouse is at a 90° angle to the ship's path. What is the distance between the ship and the lighthouse at this second sighting? (h)

When there are no visible landmarks, sailors at sea depend on the location of stars or the Sun for navigation. For example, in the Northern Hemisphere, Polaris (the North Star), stays approximately at the same angle above the horizon for a given latitude. If Polaris appears higher overhead or closer to the horizon, sailors can tell whether their course is taking them north or south.

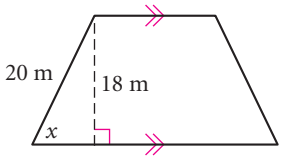
This painting by Winslow Homer (1836–1910) is titled *Breezing Up* (1876).

For Exercises 8–16, find each length or angle measure accurate to the nearest whole unit.

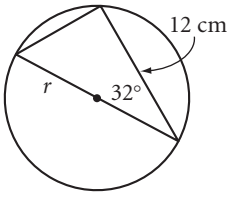
8. $a \approx$?



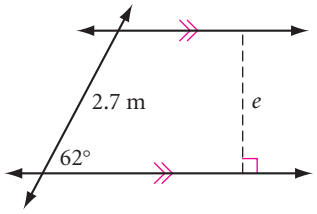
9. $x \approx$?



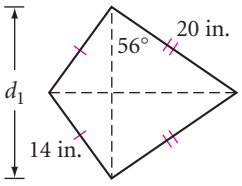
10. $r \approx$?



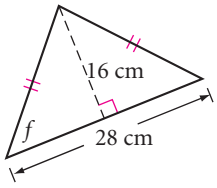
11. $e \approx$?



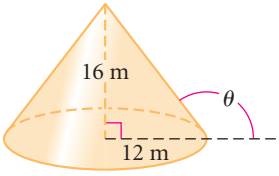
12. $d_1 \approx$? (h)



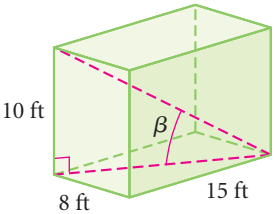
13. $f \approx$?



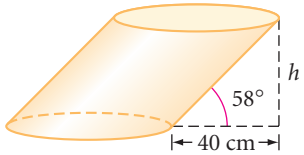
14. $\theta \approx$?



15. $\beta \approx$? (h)



16. $h \approx$?



Review

For Exercises 17–19, find the measure of each angle to the nearest degree.

17. $\sin D = 0.7071$ **18.** $\tan E = 1.7321$ **19.** $\cos F = 0.5$

Technology CONNECTION

The earliest known navigation tool was used by the Polynesians, yet it didn't measure angles. Early Polynesians carried several different-length hooks made from split bamboo and shells. A navigator held a hook at arm's length, positioned the bottom of the hook on the horizon, and sighted the North Star through the top of the hook. The length of the hook indicated the navigator's approximate latitude. Can you use trigonometry to explain how this method works?

20. Solve for x.

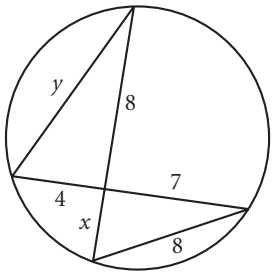
a. $4.7 = \frac{x}{3.2}$

b. $8 = \frac{16.4}{x}$

c. $0.3736 = \frac{x}{14}$

d. $0.9455 = \frac{2.5}{x}$

21. Find x and y.



22. A 3-by-5-by-6 cm block of wood is dropped into a cylindrical container of water with radius 5 cm. The level of the water rises 0.8 cm. Does the block sink or float? Explain how you know.

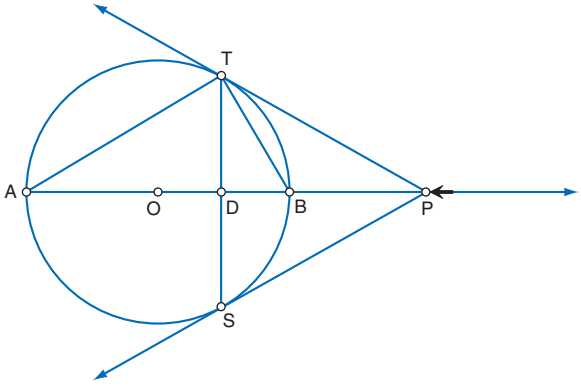
23. Scalene triangle ABC has altitudes $\overline{AX}$, $\overline{BY}$, and $\overline{CZ}$. If $AB > BC > AC$, write an inequality that relates the heights.

24. In the diagram at right, $\overrightarrow{PT}$ and $\overrightarrow{PS}$ are tangent to circle O at points T and S, respectively. As point P moves to the right along $\overrightarrow{AB}$, describe what happens to each of these measures or ratios.

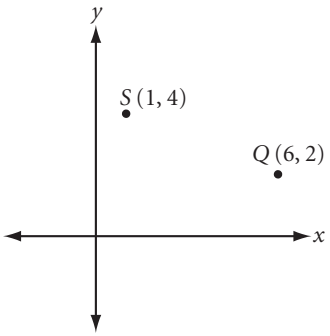
a. $m\angle TPS$ **b.** OD

c. $m\angle ATB$ **d.** Area of $\triangle ATB$

e. $\frac{AP}{BP}$ **f.** $\frac{AD}{BD}$



25. Points S and Q, shown at right, are consecutive vertices of square $SQRE$. Find coordinates for the other two vertices, R and E. There are two possible answers. Try to find both.



project

LIGHT FOR ALL SEASONS

You have seen that roof design is a practical application of slope—steep roofs shed snow and rain. But have you thought about the overhang of a roof?

In a hot climate, a deep overhang shelters windows from the sun.

In a cold climate, a narrow overhang lets in more light and warmth.

What roof design is common for homes in your area? What factors would an architect consider in the design of a roof relative to the position, size, and orientation of the windows? Do some research and build a shoebox model of the roof design you select.

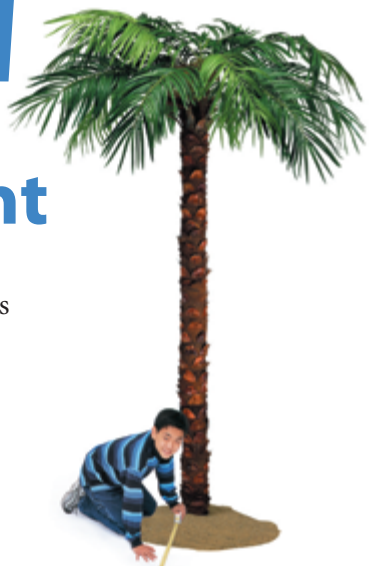
What design is best for your area will depend on your latitude, because that determines the angle of the sun's light in different seasons. Research the astronomy of solar angles, then use trigonometry and a movable light source to illustrate the effects on your model.

Your project should include

- Research notes on seasonal solar angles.
- A narrative explanation, with mathematical support, for your choice of roof design, roof overhang, and window placement.
- Detailed, labeled drawings showing the range of light admitted from season to season, at a given time of day.
- A model with a movable light source.

Exploration

Indirect Measurement



In Chapter 11, you used shadows, mirrors, and similar triangles to measure the height of tall objects that you couldn't measure directly. Right triangle trigonometry gives you yet another method of indirect measurement.

In this exploration, you will use two or three different methods of indirect measurement. Then you will compare your results from each method.

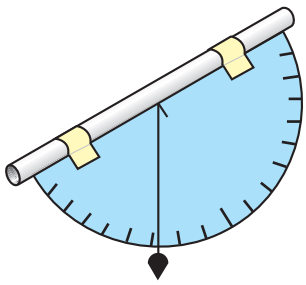
Activity

Using a Clinometer

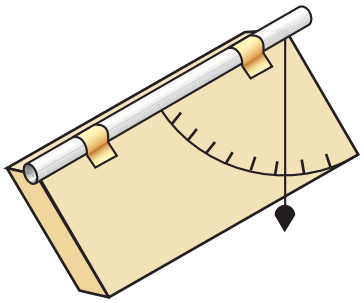
You will need

- a measuring tape or metersticks
- a clinometer (use the Making a Clinometer worksheet or make one of your own design)
- a mirror

In this activity, you will use a **clinometer**—a protractor-like tool used to measure angles. You probably will want to make your clinometer in advance, based on one of the designs below. Practice using it before starting the activity.



Clinometer 1



Clinometer 2

Step 1 Locate a tall object that would be difficult to measure directly. Start a table like this one.

Name of object	Viewing angle	Height of observer's eye	Distance from observer to object	Calculated height of object

Step 2 Use your clinometer to measure the viewing angle from the horizontal to the top of the object.

Step 3 Measure the observer's eye height. Measure the distance from the observer to the base of the object.

Step 4 Calculate the approximate height of the object.

U.S. Forest Service Ranger Al Sousi uses a clinometer to measure the angle of a mountain slope. In snowy conditions, a slope steeper than 35° can be a high avalanche hazard.

Step 5 Use either the shadow method or the mirror method or both to measure the height of the same object. How do your results compare? If you got different results, explain what part of each process could contribute to the differences.

Step 6 Repeat Steps 1–5 for another tall object. If you measure the height of the same object as another group, compare your results when you finish.

IMPROVING YOUR VISUAL THINKING SKILLS

Puzzle Shapes

Make five of these shapes and assemble them to form a square. Does it take three, four, or five of the shapes to make a square?